Structures and Stiffness

ENGR 10
Introduction to Engineering
The support structure should be optimized for weight and stiffness (deflection)
Wind Turbine Structure

Hollow tapered tube

Lattice structure

Hollow tube with guy wire
Wind Turbine Structure

Structural support

Tube with guy wire and winch

Tripod support
Wind Turbine Structure

World Trade Center in Bahrain

Three giant wind turbine provides 15% of the power needed.
Support structure failure, New York. Stress at the base of the support tower exceeding the strength of the material.
Support structure failure, Denmark. Caused by high wind
Blade failure, Illinois. Failure at the thin section of the blade.

Support structure failure, UK

Lightning strike, Germany.
Many different forms
Foam Board

Recycled Materials
Spring Stiffness

where

\[ F = k \Delta x \]

where

- \( k \) = spring constant
- \( \Delta x \) = spring stretch
- \( F \) = applied force
**Stiffness (Spring)**

- Deflection is proportional to load, \( F = k (\Delta x) \)

\[
k = \frac{\text{load}}{\text{deflection}}
\]

Slope of Load-Deflection curve:

The “Stiffness”
Stiffness (Solid Bar)

- Stiffness in tension and compression
  - Applied Forces $F$, length $L$, cross-sectional area, $A$, and material property, $E$ (Young’s modulus)

$$k = \frac{F}{\delta}$$

$$\delta = \frac{FL}{AE}$$

$$k = \frac{AE}{L}$$

Stiffness for components in tension-compression

$E$ is constant for a given material

$E$ (steel) = $30 \times 10^6$ psi

$E$ (Al) = $10 \times 10^6$ psi

$E$ (concrete) = $3.4 \times 10^3$ psi

$E$ (Kevlar, plastic) = $19 \times 10^3$ psi

$E$ (rubber) = 100 psi
Stiffness

- Stiffness in bending

- How does the material resist the applied load?
  - Think about what happens to the material as the beam bends
    - Inner “fibers” (A) are in compression
    - Outer “fibers” (B) are in tension
**Stiffness of a Cantilever Beam**

Deflection of a Cantilever Beam

\[ Y = \text{deflection} = \frac{FL^3}{3EI} \]

- **F** = force
- **L** = length
- **E** = modulus of elasticity
- **I** = moment of inertia

**Fixed end**

\[ k = \frac{F}{Y} = \frac{3EI}{L^3} \]
Concept of Area Moment of Inertia

Deflection of a Cantilever Beam

\[ Y = \text{deflection} = \frac{FL^3}{3EI} \]

\( F = \text{force} \)

\( L = \text{length} \)

Mathematically, the area moment of inertia appears in the denominator of the deflection equation, therefore;

The larger the area moment of inertia, the less a structure deflects (greater stiffness)

\[ k = \frac{F}{Y} = \frac{3EI}{L^3} \]
Clicker Question

kg is a unit of force

A) True
B) False
All 3 springs have the same initial length. Three springs are each loaded with the same force $F$. Which spring has the greatest stiffness?

A. $K_1$
B. $K_2$
C. $K_3$
D. They are all the same
E. I don’t know
Default is:
- first column plots on x axis
- second column plots on y axis

Note:
- Intercept = 0

The equation is:
\[ y = 0.6364x \]
\[ R^2 = 0.9489 \]
**Concept of Area Moment of Inertia**

The *Area Moment of Inertia* is an important parameter in determine the state of stress in a part (component, structure), the resistance to buckling, and the amount of *deflection in a beam*.

The area moment of inertia allows you to tell how stiff a structure is.

The *Area Moment of Inertia*, $I$, is a term used to describe the capacity of a cross-section (profile) to resist bending. It is always considered with respect to a reference axis, in the $X$ or $Y$ direction. It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis. The reference axis is usually a centroidal axis.
Mathematical Equation for Area Moment of Inertia

\[ I_{xx} = \sum (A_i) (y_i)^2 = A_1(y_1)^2 + A_2(y_2)^2 + \ldots A_n(y_n)^2 \]

\[ A \text{ (total area)} = A_1 + A_2 + \ldots A_n \]
Moment of Inertia – Comparison

1

Load

2 x 8 beam

I_1

Maximum distance of 1 inch to the centroid

I_2

Maximum distance of 4 inch to the centroid

2 x 8 beam

I_2 > I_1, orientation 2 deflects less

Same load and location
Moment of Inertia Equations for Selected Profiles

- **Round solid section**
  \[ I = \frac{\pi (d)^4}{64} \]

- **Round hollow section**
  \[ I = \frac{\pi}{64} [(d_o)^4 - (d_i)^4] \]

- **Rectangular solid section**
  \[ I = \frac{1}{12} bh^3 \]

- **Rectangular hollow section**
  \[ I = \frac{1}{12} BH^3 - \frac{1}{12} bh^3 \]
Show of Hands

• A designer is considering two cross sections as shown. Which will produce a stiffer structure?

A. Solid section
B. Hollow section
C. I don’t know

hollow rectangular section 2.25” wide X 1.25” high X .125” thick

B = 2.25”, H = 1.25”

b = 2.0”, h = 1.0”
Example – Optimization for Weight & Stiffness

Consider a solid rectangular section 2.0 inch wide by 1.0 high.

\[ I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(1)^3 = 0.1667, \text{ Area} = 2 \]

Now, consider a hollow rectangular section 2.25 inch wide by 1.25 high by 0.125 thick.

\[ I = \frac{1}{12}bh^3 = \frac{1}{12}(2.25)(1.25)^3 - \frac{1}{12}(2)(1)^3 = 0.3662 - 0.1667 = 0.1995 \]

Area = 2.25x1.25 – 2x1 = 0.8125

\[ \frac{(0.1995 - 0.1667)}{0.1667} \times 100 = 0.20 = 20\% \text{ less deflection} \]

Compare the weight of the two parts (same material and length), so only the cross sectional areas need to be compared.

\[ \frac{(2 - 0.8125)}{2} = 0.6 = 60\% \text{ lighter} \]

So, for a slightly larger outside dimension section, 2.25x1.25 instead of 2 x 1, you can design a beam that is **20% stiffer and 60% lighter**
The plot shows load versus deflection for three structures. Which is stiffest?

A. A  
B. B  
C. C  
D. I don’t know
Stiffness Comparisons for Different Sections

Graph of load against displacement for several different geometries of brass beams

Load / kg

Displacement / $10^{-3}$ m

Stiffness = slope

Square
Box
Rectangular Horizontal
Rectangular Vertical
Material and Stiffness

E = Elasticity Module, a measure of material deformation under a load.

Deflection of a Cantilever Beam

\[ Y = \text{deflection} = \frac{FL^3}{3EI} \]

The higher the value of E, the less a structure deflects (higher stiffness)
Material Strength

Standard Tensile Test

Standard Specimen

Ductile Steel (low carbon)

$S_y$ – yield strength

$S_u$ – fracture strength

$\sigma$ (stress) = Load / Area

$\varepsilon$ (strain) = (change in length) / (original length)
Common Mechanical Properties

• Yield Strength \((S_y)\) – the highest stress a material can withstand and still return exactly to its original size when unloaded.

• Ultimate Strength \((S_u)\) - the greatest stress a material can withstand, fracture stress.

• Modulus of elasticity \((E)\) - the slope of the straight portion of the stress-strain curve.

• Ductility - the extent of plastic deformation that a material undergoes before fracture, measured as a percent elongation of a material.

\[
\%\text{ elongation} = \frac{\text{final length, at fracture} - \text{original length}}{\text{original length}}
\]

• Resilience - the capacity of a material to absorb energy within the elastic zone (area under the stress-strain curve in the elastic zone)

• Toughness - the total capacity of a material to absorb energy without fracture (total area under the stress-strain curve)
Steel is 3 times stiffer than Aluminum and 100 times stiffer than Plastics.
Plastic is 7 times lighter than steel and 3 times lighter than aluminum.
Impact of Structural Elements on Overall Stiffness

Rectangle deforms

Triangle rigid
Clicker Question

The higher the Modulus of Elasticity (E), the lower the stiffness

A. True
B. False
Clicker Question

Which of the following materials is the stiffest?

A. Cast Iron  
B. Aluminum  
C. Polycarbonate  
D. Steel  
E. Fiberglass
Clicker Question

The applied load affects the stiffness of a structure.

A. True
B. False
Stiffness Testing
Stiffness Testing Apparatus

Load pulling on tower

Dial gage to measure deflection

Successful testers

weights